**ASSESSMENT 8**

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| **Date:** | 26-05-2020 | **Name:** | Sheela Golasangi |
| **Course:** | Digital Signal Processing | **USN:** | 4AL16EC068 |
| **Topic:** | Gibbs phenomena using python, Forier transform derivatives, laplace transform of first order, applications of Z transform | **Semester & Section:** | VIII  ‘B’ |
| **Github Repository:** | Sheela Golasangi |  |  |

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| **FORENOON SESSION DETAILS** |
| **Image of session** |
| **REPORT**  **FOURIER SERIES AND GIBBS PHENOMENA**  The Gibbs phenomenon, discovered by [Henry Wilbraham](https://en.wikipedia.org/wiki/Henry_Wilbraham) ([1848](https://en.wikipedia.org/wiki/Gibbs_phenomenon#CITEREFWilbraham1848)) and rediscovered by [J. Willard Gibbs](https://en.wikipedia.org/wiki/Willard_Gibbs) ([1899](https://en.wikipedia.org/wiki/Gibbs_phenomenon#CITEREFGibbs1899)), is the peculiar manner in which the [Fourier series](https://en.wikipedia.org/wiki/Fourier_series) of a [piecewise](https://en.wikipedia.org/wiki/Piecewise) continuously differentiable [periodic function](https://en.wikipedia.org/wiki/Periodic_function) behaves at a [jump discontinuity](https://en.wikipedia.org/wiki/Jump_discontinuity). The nth [partial sum](https://en.wikipedia.org/wiki/Partial_sum) of the Fourier series has large oscillations near the jump, which might increase the maximum of the partial sum above that of the function itself. The overshoot does not die out as n increases, but approaches a finite limit.[[3]](https://en.wikipedia.org/wiki/Gibbs_phenomenon#cite_note-Carslaw-3) This sort of behavior was also observed by experimental physicists, but was believed to be due to imperfections in the measuring apparatus.This is one cause of [ringing artifacts](https://en.wikipedia.org/wiki/Ringing_artifacts) in [signal processing](https://en.wikipedia.org/wiki/Signal_processing).  Informally, the Gibbs phenomenon reflects the difficulty inherent in approximating a [discontinuous function](https://en.wikipedia.org/wiki/Discontinuous_function) by a finite series of [continuous](https://en.wikipedia.org/wiki/Continuous_function) sine and cosine waves. It is important to put emphasis on the word finite because even though every partial sum of the Fourier series overshoots the function it is approximating, the limit of the partial sums does not. The value of x where the maximum overshoot is achieved moves closer and closer to the discontinuity as the number of terms summed increases so, again informally, once the overshoot has passed by a particular x, convergence at that value of x is possible.  There is no contradiction in the overshoot converging to a non-zero amount, but the limit of the partial sums having no overshoot, because the location of that overshoot moves. We have [pointwise convergence](https://en.wikipedia.org/wiki/Pointwise_convergence), but not [uniform convergence](https://en.wikipedia.org/wiki/Uniform_convergence). For a piecewise C1 function the Fourier series converges to the function at every point except at the jump discontinuities. At the jump discontinuities themselves the limit will converge to the average of the values of the function on either side of the jump. This is a consequence of the [Dirichlet theorem](https://en.wikipedia.org/wiki/Dirichlet_conditions).  The Gibbs phenomenon is also closely related to the principle that the decay of the Fourier coefficients of a function at infinity is controlled by the smoothness of that function; very smooth functions will have very rapidly decaying Fourier coefficients (resulting in the rapid convergence of the Fourier series), whereas discontinuous functions will have very slowly decaying Fourier coefficients (causing the Fourier series to converge very slowly).  Note for instance that the Fourier coefficients 1, −1/3, 1/5, ... of the discontinuous square wave described above decay only as fast as the [harmonic series](https://en.wikipedia.org/wiki/Harmonic_series_(mathematics)), which is not [absolutely convergent](https://en.wikipedia.org/wiki/Absolutely_convergent); indeed, the above Fourier series turns out to be only conditionally convergent for [almost every](https://en.wikipedia.org/wiki/Almost_everywhere) value of x. This provides a partial explanation of the Gibbs phenomenon, since Fourier series with absolutely convergent Fourier coefficients would be [uniformly convergent](https://en.wikipedia.org/wiki/Uniformly_convergent) by the [Weierstrass M-test](https://en.wikipedia.org/wiki/Weierstrass_M-test" \o "Weierstrass M-test) and would thus be unable to exhibit the above oscillatory behavior.  By the same token, it is impossible for a discontinuous function to have absolutely convergent Fourier coefficients, since the function would thus be the uniform limit of continuous functions and therefore be continuous, a contradiction. See [more about absolute convergence of Fourier series](https://en.wikipedia.org/wiki/Convergence_of_Fourier_series#Absolute_convergence).  **FOURIER TRANSFORM DERIVATIVE.**  It uses the time as a function of frequency in many physics applications. In mathematical terms, we can express the Fourier Transform ‘h ’ on a function that can be integrated and has the Domain and range from real to constant Numbers. This is considered for each and every real number.  The Laplace Transform can be used to solve differential equations using a four step process.   1. Take the Laplace Transform of the differential equation using the [derivative property](https://lpsa.swarthmore.edu/LaplaceXform/FwdLaplace/LaplaceProps.html#First_) (and, perhaps, others) as necessary. 2. Put initial conditions into the resulting equation. 3. Solve for the output variable. 4. Get result from [Laplace Transform tables](https://lpsa.swarthmore.edu/LaplaceZTable/LaplaceZFuncTable.html).  If the result is in a form that is not in the tables, you'll need to use the[Inverse Laplace Transform](https://lpsa.swarthmore.edu/LaplaceXform/InvLaplace/InvLaplaceXform.html).   **APPLICATIONS OF Z TRANSFORM**  Z transform is used in many applications of mathematics and signal processing. Thelists of applications of z transform are:- -Uses to analysis of digital filters. -Used to simulate the continuous systems. -Analyze the linear discrete system. -Used to finding frequency response. -Analysis of discrete signal. -Helps in system design and analysis and also checks the systems stability. -For automatic controls in telecommunication. -Enhance the electrical and mechanical energy to provide dynamic nature of the system. If we see the main applications of z transform than we find that it is analysis tool that analyze the whole discrete time signals and systems and their related issues. If we talk the application areas of This transform wherever it is used, they are:- -Digital signal processing. -Population science. -Control theory. -Digital signal processing.  **LAPLACE TRANSFORM**  The Laplace transform, named after its inventor [Pierre-Simon Laplace](https://en.wikipedia.org/wiki/Pierre-Simon_Laplace) ([/ləˈplɑːs/](https://en.wikipedia.org/wiki/Help:IPA/English)), is an integral transform that converts a function of a real variable {\displaystyle t} (often time) to a function of a [complex variable](https://en.wikipedia.org/wiki/Complex_analysis) {\displaystyle s} ([complex frequency](https://en.wikipedia.org/wiki/Complex_frequency)). The transform has many applications in science and engineering because it is a tool for solving [differential equations](https://en.wikipedia.org/wiki/Differential_equation). In particular, it transforms differential equations into algebraic equations and [convolution](https://en.wikipedia.org/wiki/Convolution) into multiplication. |